

Zeno's Paradox – Theorem & Proof

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Abstract

In an attempt to resolve the 2500 year old, Zeno's paradox, this paper finds that the only real numbers are 1 and 0; all other whole numbers are assumed to be irrational numbers. The number one represents the starting point (10 feet) and zero represents the ending point (the ground). All half numbers are considered irrational and declining in size. Each subsequent half is an irrational approximation; each iteration is incrementally smaller (1, 1.999, 2.998, 3.997). Therefore, the falling stone eventually reaches zero, resolving Zeno's paradox.

Zeno's Paradox – Theorem & Proof

Zeno's paradoxes are attributed to a Greek philosopher, Zeno of Elea. For more than 2500 years, a solution to the key paradox has eluded schoolchildren and mathematicians alike. Simply put, Zeno's paradox claims that it is impossible to travel from point A to point B, since an infinite number of halves exist between the two points. For instance, a rock cannot fall from a height of 10 feet to the ground of zero feet, because the first half is five feet, the next half is 2.5 feet and then 1.25 feet ad infinitum. Thus the rock never reaches terra firma. An answer to the riddle may be found in a subtle yet important ambiguity within the fundamentals of mathematics. Building upon Kurt Gödel's incompleteness theorem as well as the set theory work of Georg Cantor, this paper includes a novel theorem and proof regarding Zeno's paradox (©Copyright Chris Waltzek, 2013) (Gödel, 1992).

If one assumes that real numbers / rational numbers (0, 1, 2, 3...) are based on empirical representations, then an interesting conundrum exists. For example, if one finger on a human hand represents the number one, then common sense suggests that the remaining 7 digits result with a total of eight fingers. However, none of the 7 additional fingers are a perfect emulation of the first finger. Thus if one accepts this highly stringent, yet empirical definition of a real number, then the finger two is merely an irrational representation of the number one. This leads to a new number sequence: 1, 1.999, 2.998, 3.997... Even at the subatomic level, the Heisenberg uncertainty principle suggests that no single atom exists at any given time in precisely the same state (protons, neutrons and electrons gyrate). Therefore, it is the assertion of this paper that the only real numbers that exist are the numbers 1 and 0 and the remaining numbers are irrational facsimiles. For example, sunrise and sunset, on and off, hot and cold; the whole of existence is

assumed to be binary, punctuated with continuous irrational pauses. Ergo, the numbers 2 and 3 exist, but only as an estimated value based on 1, such as: 1.99 and 2.97.

If one accepts the hypothesis that the only real numbers are the beginning point and the ending point, represented by 1 and 0, then Zeno's paradox is easily solvable. Let the starting point (10 feet) represent the number 1 and the ending point (the ground) represent the real number zero. The axiom suggests that half numbers are irrational and mere representations, so when the falling stone reaches 5 feet, it actually reaches 4.999... or some approximation. Each subsequent half is an irrational approximation; each iteration is incrementally smaller (1, 1.999, 2.998, 3.997). Thus, the stone *must* eventually reach zero, i.e. the ground at an accelerating rate, as every schoolchild knows intuitively. The following humble theorem and proof would undoubtedly delighted Bertrand Russell and Ludwig Wittgenstein.

Theorem (©Copyright Chris Waltzek, 2013): the only real numbers are 1 and 0; all other numbers are irrational numbers. A falling stone must eventually reach zero at an accelerating rate, solving Zeno's paradox.

Proof: Let the number one represent the starting point (10 feet) and zero represent the ending point (the ground). All half numbers are considered irrational and declining in size. When the falling stone reaches 5 feet, it actually reaches 4.999... or some approximation. Each subsequent half is an irrational approximation; each iteration is incrementally smaller (1, 1.999, 2.998, 3.997). The falling stone eventually reach zero. □

Discussion

This paper finds that the only real numbers are 1 and 0; all other numbers are irrational numbers. The number one represents the starting point (10 feet) and zero represents the ending point (the ground). All half numbers are considered irrational and declining in size. When the

falling stone reaches 5 feet, it actually reaches 4.999... or some approximation. Each subsequent half is an irrational approximation; each iteration is incrementally smaller (1, 1.999, 2.998, 3.997). The falling stone eventually reach zero, resolving Zeno's paradox. Future research stemming from this paper could include the obvious parallels between the theorem and the accelerating nature of gravity. Chaos theorists and fractal researchers might find useful the concept of only two real numbers, 1 and 0.

References

Gödel, K. 1992. On formally undecidable propositions of principia mathematica and related systems. Dover books on mathematics.